

Testing of Hypothesis

Student's t-test (Test of Significance for Single Population Mean, σ Unknown)

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Example :

A company packages a food product and claims each packet weighs 500 grams on average. A random sample of 10 packets was weighed (in grams) and the following data was obtained. Test, at 5% level of significance, whether the mean weight of the packets is 500 grams.

478, 485, 490, 495, 500, 505, 512, 518, 522, 530

Solution:

Let the hypothesized value of μ be $\mu_0 = 500$. Hence the set of hypotheses is

$$H_0: \mu = 500$$

against $H_a: \mu \neq 500$

at α level of significance.

The test statistic is

$$t = \frac{|\bar{x} - \mu|}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$$

Where

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \quad s = \sqrt{\frac{1}{n-1} \left\{ \sum_{i=1}^n d_i^2 - \frac{1}{n} \left(\sum_{i=1}^n d_i \right)^2 \right\}}$$

Table for calculation of mean and sample mean square

Weight (x_i)	$d_i = x_i - \bar{x}$ (503.5)	d_i^2
478	-25.5	650.25
485	-18.5	342.25
490	-13.5	182.25
495	-8.5	72.25
500	-3.5	12.25
505	1.5	2.25
512	8.5	72.25
518	14.5	210.25
522	18.5	342.25
530	26.5	702.25
5035	0	2588.5

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{35}{10} = 503.5$$

$$s = \sqrt{\frac{1}{n-1} \left\{ \sum_{i=1}^n d_i^2 - \frac{1}{n} \left(\sum_{i=1}^n d_i \right)^2 \right\}}$$

$$s = \sqrt{\frac{1}{10-1} \left\{ 2588.5 - \frac{1}{10} (0)^2 \right\}}$$

$$s^2 = 287.61$$

$$s = \sqrt{287.61} = 16.96$$

Calculated value of test statistic

$$t = \frac{|\bar{x} - \mu|}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$$

$$t = \frac{|503.5 - 500|}{\frac{16.96}{\sqrt{10}}}$$

$$t = 3.5 / 5.363 = \mathbf{0.653}$$

Decision rule:

Based on the t -table for $df = (n-1) = 9$ and $\alpha = 0.05$, we have the critical value $(t_{9, (\alpha/2)}) = t_{9, 0.025} = 2.26$ (t_{tab}). Since $t_{\text{cal}} (0.653) < t_{\text{tab}} (2.26)$, we do not reject H_0 at 5% level of significance and conclude that the mean weight of the packets is 500 grams — the company's claim is supported by the data.

Do your self

Example 1: The recommended average wheat yield is 4.5 t/ha. A scientist collected data from 10 farms.

Farm	Yield (t/ha)
1	4.8
2	4.4
3	4.6
4	4.7
5	4.3
6	4.9
7	4.5
8	4.6
9	4.8
10	4.7

Example 2: Recommended soil organic carbon = 0.80%

Field	Organic Carbon (%)
1	0.82
2	0.76
3	0.79
4	0.81
5	0.77
6	0.84
7	0.80
8	0.78
9	0.83
10	0.79